A Simple Model For Estimating End Of Construction Pore Pressures: Part 1 - Foundation Pore Pressures[1]

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INTRODUCTION

When a highly saturated compressible soil of low permeability is loaded, the fluid present in the soil's pore space inhibits the volumetric strain necessary to transfer the load to the soil skeleton, and this phenomenon manifests itself in the development of excess pore pressures in the pore fluid of the soil. The excess pore water pressures generated in this way dissipate over time; however, they can have an adverse effect on the stability of the structure both during and immediately subsequent to construction. Consequently, it is important for the designer to be able to estimate the magnitude of these pore pressures at any point during construction so the data can be incorporated into conventional slope stability analyses. In addition, the ability to estimate the rate at which excess pore pressures dissipate enables the designer to develop a plan for staged construction of the structure if required by the specific site conditions. While the problem of construction generated pore pressures is most commonly associated with large earth structures, such as earth dams, the phenomenon is not limited to earth structures and also occurs beneath any structure, such as a spread footing, that applies a load to an underlying soil foundation. Furthermore, the soil need not be fully saturated for excess pore pressures to develop.

This is the first of two application oriented papers available over the Internet at no cost, presenting the details of an analytical model for estimating both the magnitude and rate of dissipation of construction generated excess pore pressures. The model was originally developed by the author [4] to analyze the problem of excess pore pressure development within the core zone of an earth and rockfill dam during construction, where both the distribution of stresses and dissipation of pore pressures are essentially two-dimensional. While the method has previously been presented by the author in published form [5, 6], most of us find it difficult to implement a new method without information that frequently does not survive the editorial process for papers that appear in journals and conference proceedings. The purpose of this paper is to provide what is hoped will be a sufficient amount of detail for an engineer to apply the method to his/her specific problem. The method was specifically developed to permit the analyses to be performed on a personal computer using commercially available spreadsheet software, thereby eliminating the need for special application software. As a companion to each paper a spreadsheet has also been prepared by the author, in Microsoft Excel (97-2000 & 5.0/95 compatible formats), illustrating the solution to an example problem. This spreadsheet example is available at no cost from the author's web site.

For the purpose of illustrating the basic method this first paper will be limited to the case of a fully saturated soft soil foundation beneath an earth structure. The author recommends this paper as the best place to begin for those individuals who may not be familiar with the various elements of this complex problem. The second paper discusses how the model can be extended to account for an unsaturated soil, as well as the case of a moving drainage boundary, and illustrates how to estimate construction generated pore pressures within the core of a zoned earth dam. It is the author's hope that engineers will attempt to use the methods described in these papers in concert with field instrumentation to monitor actual performance, thereby either further validating the model or providing a mechanism through which the methods can be modified and improved. The author would appreciate any feedback from individuals regarding the methods presented in these papers and would be willing to answer questions via e-mail to assist those developing a solution to their specific problem.

ELEMENTS OF THE MODEL AND EXAMPLE PROBLEM

The problem of estimating construction generated excess pore pressures in soil can be thought of in terms of three interdependent components. The first involves establishing the stresses applied to the soil. The second involves defining the pore pressure response of the soil to the stress changes that occur as the load is applied, which is primarily a function of the stress-strain properties and initial degree of saturation of the soil. The third is the consolidation process, which addresses the dissipation of excess pore pressures during construction.

The primary components of the model consist of:

a) an approximate method for estimating the stresses beneath the loaded area,
b) a method for computing the pore pressures generated in the foundation, and
c) a two-dimensional finite difference consolidation algorithm using the uncoupled multi-dimensional theory of Terzaghi-Rendulic to account for pore pressure dissipation during construction.

The various elements of the model will be discussed in subsequent paragraphs and the algorithms are illustrated in a spreadsheet that presents the solution to an example problem for the case of construction generated excess pore pressures in the foundation beneath an earth embankment. As previously noted, the spreadsheet solution is available at no cost over the Internet in Microsoft Excel format. The example problem is described below and illustrated in Figure 1.

An embankment is to be constructed over a compressible soil foundation. The ground along the embankment alignment slopes downward uniformly at 3.0(H):1(V) from Sta. 0+00 at Elev. 100 feet to Sta. 1+80 at Elev. 40 feet, remains uniform at Elev. 40 feet between Sta. 1+80 and Sta. 5+80, and slopes upward at 4.0(H):1(V) from Sta. 5+80 at Elev. 40 feet to Sta. 8+20 at Elev. 100 feet. The embankment will have a maximum height of 60 feet from Sta. 1+80 to Sta. 5+80, side slopes of 2.5(H):1(V), and a width of 100 feet at the top (Elev. 100). The embankment fill will have a moist unit weight of 130 pcf (pounds per cubic foot). The soil foundation beneath the embankment consists of a soft, low permeability soil, with a consolidation coefficient (c_v) of 0.80 square feet per day, and extends to a depth of several hundred feet below the ground surface. For the purpose of this problem, the foundation will be assumed to be fully saturated, and the foundation soil will have a saturated unit weight of 115 pcf.

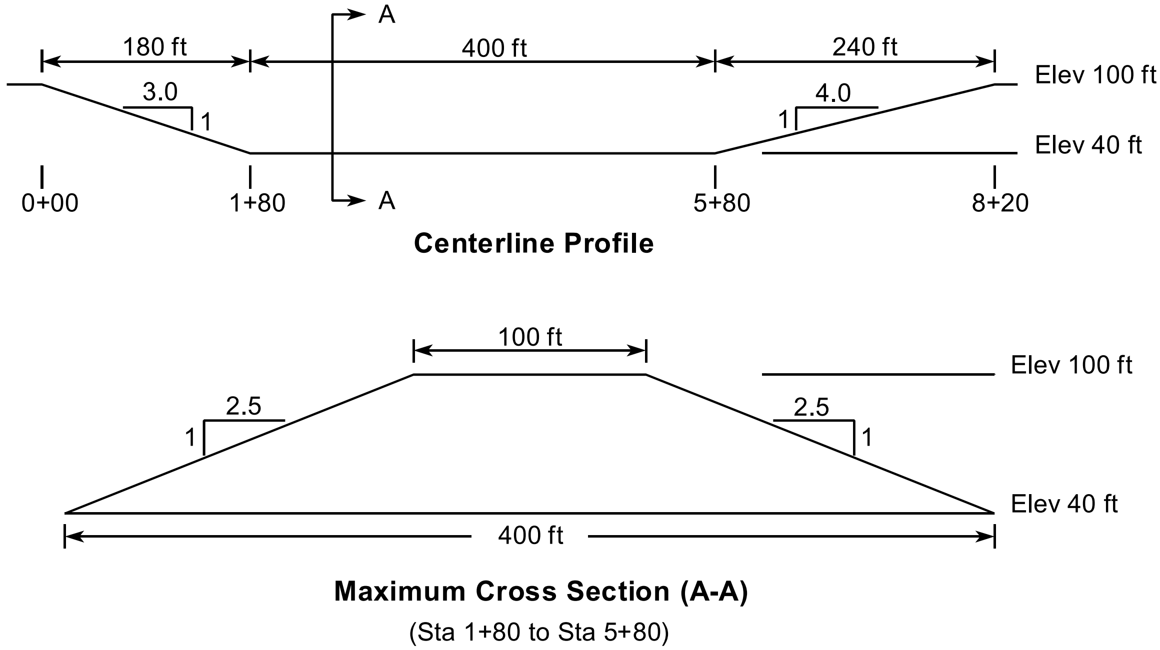

FIGURE 1: Centerline profile and maximum embankment cross section for Problem 1.

STRESS DISTRIBUTION MODEL

Predicting the stress changes in the foundation beneath a surface load is extremely difficult due to the complex stress-strain behavior of soil and the changing physical properties of the soil during the consolidation process, not to mention the fact that we are rarely so fortunate as to encounter a truly uniform, homogeneous geologic deposit. Consequently, even the most rigorous analyses may yield results that can only be considered to be an estimate of the actual values. The method proposed herein uses a model based on the theory of elasticity. Despite the fact that soil behavior is not entirely elastic, elastic solutions are frequently used to estimate stresses in an earth mass. The rationale for using elastic solutions is that reasonable agreement has been observed between measured and predicted stresses where the boundary conditions of the solution approximate the field conditions [11].

The model assumes that the stresses in the foundation beneath the embankment fill are distributed in the same manner as that of a uniform strip load on a semi-infinite linearly elastic isotropic foundation. The total vertical, horizontal and shear stresses appropriate to a uniform strip load on a semi-infinite linearly elastic isotropic foundation are given in Equation 1 through 3, and for plane strain conditions are simple functions of geometry and the vertical stress increase (w) imposed by the uniform strip load [9]. Note that all angles in Equations 1 through 3 are in radians.

$$\sigma_z = (w/\pi)[\alpha + \sin\alpha\cos(\alpha + 2\beta)] \quad (1)$$
$$\sigma_x = (w/\pi)[\alpha - \sin\alpha\cos(\alpha + 2\beta)] \quad (2)$$
$$\tau_{xz} = (w/\pi)[\sin\alpha\sin(\alpha + 2\beta)] \quad (3)$$

Figure 2 illustrates how the terms in Equations 1 through 3 are defined. A different algorithm would naturally have to be applied to the model for a case where the surface load cannot be represented by a uniform strip load, such as in the case of a rectangular spread footing.

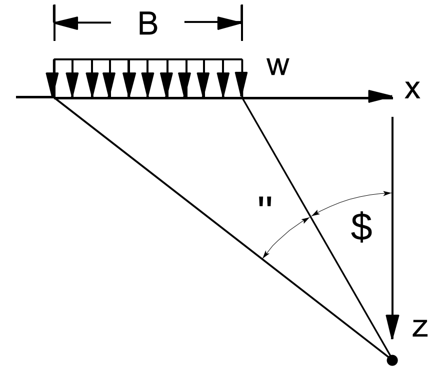

FIGURE 2: Reference system for stress distribution calculations.

Using this method an embankment fill can then be modeled as a series of successive uniform strip loads stacked or layered to simulate the embankment cross section, as illustrated in Figure 3. Each uniform strip load can be sized so as to represent a construction lift, the horizontal dimension of which decreases as a function of the embankment height and side slopes. The w term in Figure 2 is then computed simply as the product of the lift thickness and unit weight of the embankment fill material. The rationale behind this approach is that it is computationally simple and yields results that correlate well with field data from a well instrumented large dam [4].

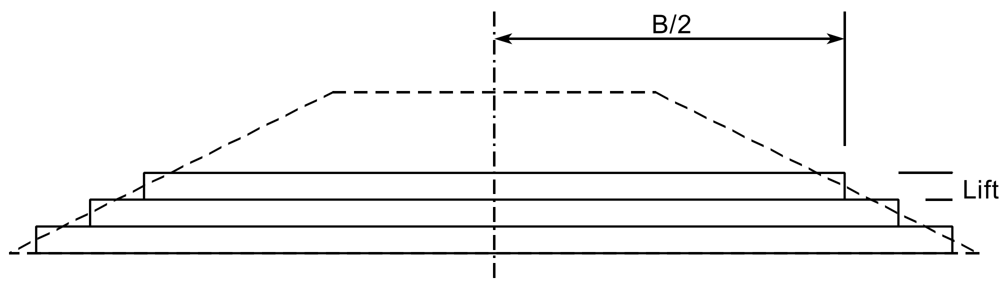

FIGURE 3: Approximation for simple elastic embankment model.

PORE PRESSURE RESPONSE IN SOIL

It has long been recognized that the excess pore pressure generated in a soil during undrained loading is a function of the total stress changes produced by the load, the soil porosity and the relative compressibilities of the soil skeleton and pore fluid. Skempton [10] developed an expression for the pore pressure change in terms of the major and minor principal stress changes and two empirical coefficients, A and B, which is perhaps the relationship most widely recognized among geotechnical engineers today. Skempton demonstrated that the B parameter is a function of the soil's degree of saturation and is essentially equal to one for a fully saturated soil. The A parameter accounts for the actual behavior of the soil structure. The A parameter has been found to be a function of the compressibility of the soil under hydrostatic stresses and the tendency of the soil to expand or contract in response to shear stresses. Skempton's work was extended by other researchers and the relationship adopted for use in this model is that proposed by Atkinson and Bransby [1], which expresses the pore pressure change in terms of the total stress invariants p and q and the empirical coefficients ‘a’ and ‘b’, which are fundamentally the same as Skempton's A and B parameters. Atkinson and Bransby's expression is:

$$\Delta u = b(\Delta p + a\,\Delta q) \tag{4}$$

Using Equations 1 through 3 for plane strain conditions, and taking Poisson's ratio equal to 0.5 for undrained loading of a saturated soil, the following expressions are obtained for the total stress invariants p and q, for use in computing the pore pressure increment Δu in Equation 4.

$$p = w\alpha/\pi \tag{5}$$

which is simply the mean normal total stress, and

$$q = (w/\pi)(\sin\alpha)(\sqrt{3}) \tag{6}$$

Appendix A provides some additional information for those readers who may wish to see how the stress invariants in Equations 5 and 6 are developed for plane strain conditions.

If it is then assumed that the pore pressure increment is simply a function of the degree of saturation and the change in the mean normal total stress, Equation 4 reduces to:

$$\Delta u = b\Delta p \tag{7}$$

Equation 7 implies basically that the soil behaves as an elastic material, and for such a material a shear stress increment (addressed by the stress invariant q) would have no effect on pore pressures. While research has shown that shear stresses actually do affect pore pressures in soil, the author has found that this simpler form of the expression yielded a better estimate of excess pore pressures in a soft soil compared to actual measured values than did methods that attempted to account for the ‘a’ parameter and the stress invariant q [4]. For the purpose of this first paper it will simply be assumed that $\Delta u = \Delta p$, where the value of $b = 1$ corresponds to a fully saturated soil. Based on the author’s experience, it is believed that this simplifying assumption should provide reasonable results when dealing with saturated soft soils; however, the approach needs to be modified somewhat for unsaturated soils and/or stiff soils, such as overconsolidated clays, where the stiffness of the soil matrix can also have a significant effect on the excess pore pressures generated. The second paper discusses how the model can be extended to account for partial degrees of saturation, along with the actual stress-strain properties of the soil.

As a final point it is important to recognize that Δu in the preceding equations refers to the excess pore pressure, which is only one component of the total pore pressure. The total pore pressure is the sum of the excess and the steady state pore pressures. For the example problem presented in this paper, the steady state pore pressure is simply the hydrostatic pore pressure for a phreatic surface at the surface of the ground. Therefore, for the purpose of stability analyses to determine the factor of safety of the embankment at any time during or subsequent to construction, the excess pore pressures computed from the model presented in this paper must be added to the steady state pore pressures at the same respective points in the ground to yield the total pore pressures for use in the stability analyses. Sheet 4 of the example spreadsheet illustrates how to compute pore pressure ratios (the ratio of total vertical stress to total pore pressure) for subsequent use in stability analyses.

RATE OF LOAD APPLICATION

The rate at which the load is applied needs to be defined for the analysis. For earthwork construction this is essentially the rate at which the embankment will be constructed in feet per day. To develop this estimate it is necessary to define a typical embankment cross section and a generalized profile along the embankment alignment, such as those defined in the example problem. From this information a spreadsheet can be developed to compute the incremental volume corresponding to each vertical foot of embankment. Then, knowing the volume of material that the contractor expects to be able to place each day one can compute the amount of time that will be required to construct each incremental foot of the embankment. If the contractor's production rate is not known, the author's experience has been that a typical rate of 8,000 to 10,000 cubic yards per day can be assumed for a large earthwork construction project, such as an earth dam.

For the example problem, the first foot of fill material from Elev. 40 to Elev. 41 (placed between Sta. 1+77 and Sta. 5+84) will have an average length along the alignment of 403.5 feet (at Elev. 40.5) and an average cross sectional width (corresponding to dimension B in Figures 2 and 3) of 397.5 feet. This yields a volume of 5,940 cubic yards of material required to complete the first foot of the embankment. With each additional foot of fill placed, the average length along the alignment will increase by 7 feet (for the 3(H):1(V) slope downstation and the 4(H):1(V) slope upstation), and the average cross sectional width will decrease by 5 feet (for the 2.5(H):1(V) side slopes). The last foot of fill material, from Elev. 99 to Elev. 100, will only require 3,100 cubic yards of material. Though the required volume of fill generally decreases with each additional foot, the actual working area decreases while the distance that the construction equipment must travel along the embankment alignment increases. Consequently, it typically requires about the same amount of time for the contractor to place and compact material near the top of the embankment as it does at the base. The total volume of fill required for this example is estimated to be 315,562 cubic yards, which is a relatively small quantity of earthwork. Therefore, if it is assumed that the contractor can place about 5,500 cubic yards per day, the "computed" time required to place and compact each foot of embankment fill varies from 1.08 days for the first foot of embankment fill to 0.56 days for the last foot, with an average of 0.96 days per foot. The fill placement rate (given in feet per day) is the inverse or reciprocal of the average time required to complete each vertical foot of embankment fill. The calculations are illustrated on Sheet 2 of the example spreadsheet, which also yields an estimate of the total volume of fill required for the embankment. Based on these calculations, a fill placement rate of 1 foot per day was selected for use in the example problem.

THE CONSOLIDATION ALGORITHM

Despite the limitations associated with small strain consolidation theory, the generalization of Terzaghi's one-dimensional theory to describe multidimensional consolidation, attributed to Terzaghi and Rendulic, has been found to correlate reasonably well with pore pressures measured in the core of a large zoned earth and rockfill dam [4]. In applying the equation to the example problem it is reasonable to assume that fluid flow through the foundation will occur in two dimensions (perpendicular to the axis of the embankment within the plane of the cross section). Consequently, the two-dimensional form of the consolidation equation should be used, which is:

$$c_v\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]=\frac{\partial u}{\partial t}-\frac{\partial \sigma}{\partial t} \tag{8}$$

in which c_v is the coefficient of consolidation, u is the excess pore water pressure, t is time, and σ is total stress. For the purpose of describing the change in pore pressure as a function of space, time and stress the equation is simply:

$$\frac{\partial u}{\partial t}=c_v\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]+\frac{\partial \sigma}{\partial t} \tag{9}$$

To solve this partial differential equation, the numerical method of finite differences will be used. In the finite difference method, the differential equation is converted into an algebraic equation by replacing the partial derivatives with difference quotients. Since differential equations are not the same as simple algebraic equations, the numerical method yields only an approximation to the solution (though potentially a very good approximation). To solve Equation 9 the author has chosen an explicit form of the finite difference solution [2].

$$\begin{aligned} u(x_0,y_0,t+\Delta t) &= u(x_0,y_0,t)+\frac{c_v}{(\Delta x)^2}\left[u(x_1,y_0,t)+u(x_3,y_0,t)-2u(x_0,y_0,t)\right]+ \\ &\frac{c_v}{(\Delta y)^2}\left[u(x_0,y_2,t)+u(x_0,y_4,t)-2u(x_0,y_0,t)\right]+\left[\sigma(t+\Delta t)-\sigma(t)\right] \end{aligned} \tag{10}$$

At first glance Equation 10 may appear to be even more intimidating than Equation 9; however, that is in all likelihood due to the subscript notation employed in the equation. The following paragraphs provide an explanation of the subscript notation, as well as the various terms in Equation 10.

In the finite difference solution, the excess pore pressure (u) is determined at discrete points within the foundation as a function of time; therefore, the excess pore pressure is a function of two spatial variables and time. In Equation 10, terms of the form $u(x_i, y_i, t)$ simply refer to values of the excess pore pressures at specific locations (nodal points) in the foundation defined by their x and y coordinates, at specific points in time subsequent to the start of construction. For example, referring to Figure 4, (x_0, y_0, t) corresponds to point 0 at time t. Equation 10 simply states that the value of u at any point in the consolidating zone subsequent to an incremental change (Δt) in time, can be determined from the values of u prior to the change Δt at that point and four points immediately adjacent to that point, located at increments Δy above and below, and Δx to the left and right, as illustrated in Figure 4. Thus, the determination of the values of excess pore pressure throughout the foundation during the consolidation process is a straightforward matter, provided that initial values of excess pore pressure are known throughout the zone and the values of Δx, Δy and Δt are properly chosen.

For individuals unfamiliar with the finite difference method it may be relevant to note that the points identified in Figure 4 as 0 through 4 are "relative" points. Point 0 is always the center point for which the value of excess pore pressure is being calculated subsequent to the time step, on the left side of Equation 10. Points 1 and 3 always refer to the nodal points directly to the left and right respectively, while points 2 and 4 always refer to the nodal points directly above and below respectively. In the analysis these points correspond to cells in the spreadsheet.

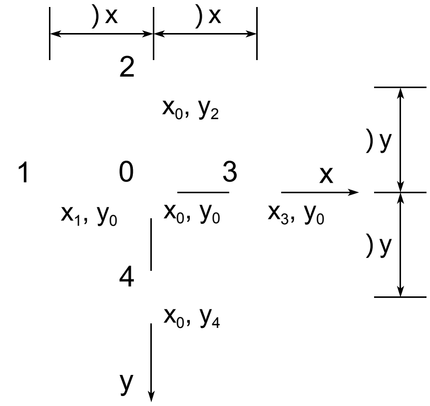

FIGURE 4: Reference system for the finite difference grid.

The advantage of the explicit form of the approximation to the governing partial differential equation is that it involves a very simple algorithm in the spreadsheet, despite the fact that it imposes certain restrictions on both the spatial and time increments in order to ensure convergence and stability of the solution. It has been shown that both convergence and stability of Equation 10 will be assured if the following criterion is satisfied [2]:

$$\frac{c_v \Delta t}{(\Delta x)^2 + (\Delta y)^2} \leq \frac{1}{8} \tag{11}$$

The second term on the right side of Equation 8 [$\partial\sigma/\partial t$] is seen in the finite difference approximation as the last term in brackets on the right side of Equation 10 [$\sigma(t + \Delta t) - \sigma(t)$], which defines the change in total stress ($\Delta\sigma$) over the time increment (Δt). The total stress change at each point in the foundation results from the application of each successive lift of embankment material. It is this total stress change that produces the excess pore pressure; therefore, the term can be interpreted as a pore pressure increment (Δu). As previously noted, if it is assumed that the pore pressure increment is simply a function of the 'b' parameter and the change in the mean normal total stress (Δp), then Equation 7 reduces to $\Delta u = \Delta p$ for a value of b = 1, corresponding to a fully saturated soil. Thus, the last term in brackets on the right side of Equation 10 [$\sigma(t + \Delta t) - \sigma(t)$] is the total stress change that produces the pore pressure increment (Δu), and for the purpose of the analysis is taken to be the mean normal total stress change (Δp) obtained using Equation 5.

DETAILS OF THE ANALYSIS

The first step in the analysis is to select an appropriate cross section and divide the foundation into a finite difference grid defined by x and y coordinates. For problems where an axis of symmetry exists, such as the centerline of the embankment, it is only necessary to perform the analysis for one side of the problem. The reason for this will become more apparent subsequently in the discussion of boundary conditions. Therefore, it will be convenient to define the x-coordinate in terms of the distance from the centerline of the embankment (centerline offset x), and the y-coordinate in terms of the depth (y) beneath the original ground surface, or elevation if the designer prefers. The points defined by this grid are the nodal points at which the values for excess pore pressure will be computed. The reader may have observed that the variable z is used to define vertical distance in

Figure 2 while the variable y is used in Figure 4 and Equation 10. There is an important reason for this that will be discussed subsequently in this section.

Within the spreadsheet, a block of columns and rows, referred to as a range, is respectively assigned centerline offsets (x) and depths (y). The specific cells within each range correspond to the nodes of the finite difference grid. While a rectangular range can be defined by the two cells that define the diagonal of the rectangle, assigning a "name" to the range will make it easier to both write and subsequently read any macros that the spreadsheet designer may create. Three ranges are needed to perform the analysis for the example problem. The information contained in each of these three ranges, along with the respective range names, is summarized below. Further details regarding these three ranges will be presented subsequently in this section.

1 - Range \u1 (Rows 23 through 43) the values of the excess pore pressure prior to the time step (which corresponds to the pore pressure terms on the right side of Equation 10),

2 - Range \u2 (Rows 52 through 72) the values of the excess pore pressure after the time step (which corresponds to the expression on the left side of Equation 10), and

3 - Range \dp (Rows 82 through 102) the values of the mean normal total stress change (Δp) due to the application of each lift of embankment material with each time step (which corresponds to the last expression on the right side of Equation 10).

In addition to naming ranges, the reader will also find it helpful to assign names to individual cells, which makes it much easier to read equations and conditional statements such as those that appear in the spreadsheet for the example problem.

The next step is to define the boundary conditions. For the two-dimensional case given by Equation 8 the designer must specify the conditions along four boundaries in the model. These boundaries can be either free draining or no flow boundaries. The results of the analysis are naturally sensitive to both the type of boundaries and their location. Note that a boundary need not be an impervious boundary to constitute a no flow boundary. An axis of symmetry, such as the centerline of the embankment, could also constitute a no flow boundary because the value of the excess pore pressure on the left side of the centerline would be equal to the value on the right side; consequently, the total head on both sides of the centerline will be equal throughout the analysis and as a result there will be no flow across the centerline. Therefore, the way to treat a no flow boundary is to modify the finite difference expression (Equation 10) for the point(s) that lie on the boundary by simply using the value of the excess pore pressure at the nodal point just inside the boundary twice in Equation 10. This is illustrated in the spreadsheet for the example problem where all of the boundaries except the surface of the foundation are treated as no flow boundaries. To assist the reader in examining the variations in the finite difference expression, in the second range (\u2) of the spreadsheet for the example problem, the cells that correspond to nodes that lie along the boundaries are shown shaded in different colors depending on the type of boundary.

Since by definition no excess pore pressures develop at a drainage boundary, the value for the excess pore pressure at all points along such boundaries is set at zero. The surface of the foundation beyond the toe of the embankment will normally be a drainage boundary; however, the foundation surface beneath the embankment may or may not be a drainage boundary, depending on the various factors governing drainage such as the relative permeabilities of the soil types. For the purpose of this example the surface of the foundation over which the embankment is placed has also been specified as a drainage boundary, which would be the case only if the permeability of the embankment material was several orders of magnitude higher than the permeability of the foundation soil. Similarly, within the foundation itself a layer of high permeability soil could also constitute a drainage boundary, provided the thickness and lateral extent of the deposit is sufficient to function as a drain. Note, however, that a lense of high permeability soil within the foundation would **not** constitute a drainage boundary since it would have no outlet or discharge point. The reader may see that it would be easy to analyze the effectiveness of vertical drains installed in the foundation to facilitate the consolidation process by simply setting the value of the excess pore pressure to zero for those nodal points at the location of the drains. However, it is important to note that since this is a two-dimensional model, which assumes that all flow occurs within the plane of the cross section, that a column of nodes set at a value of zero would constitute a drainage trench parallel to the axis of the embankment, rather than a drainage well.

For the deep homogeneous deposit in the example problem only two of the four boundaries are easily defined. As previously discussed, the centerline constitutes a no flow boundary and the surface of the foundation has been set as a drainage boundary. The remaining two boundaries, however, are not as easily defined and both their location and type have a significant effect on the results of the analysis. For example, setting the second vertical boundary (which would be parallel to the centerline) as a no flow boundary at the toe of the embankment essentially reduces the problem to one-dimensional conditions where all the flow is straight upward during excess pore pressure dissipation. Moving this boundary farther away from the toe allows for flow in that direction, which is what will actually occur in the field. The question becomes, "How far away from the first vertical boundary should the second vertical boundary be set?" The answer to this question is to set the boundary, as well as the second horizontal boundary (which would be parallel to the surface of the foundation), at positions where the results of the analysis are insensitive to their locations, and this can be established by trial and error. A place to begin would be to set the second vertical boundary at a distance beyond the embankment toe (and the second horizontal boundary at a depth) equal to the length of the base of the embankment. For the example problem the base of the embankment is 400 feet at the maximum section; therefore the lowest nodal points are located at a depth of 400 feet and the nodal points farthest away from the centerline are located at an offset of 600 feet from the centerline.

With the physical limits of the problem defined in terms of the four respective boundaries, the next step in the solution is to define an "initial condition" at each of the nodal points. The initial condition is the distribution of excess pore pressures throughout the foundation at the start of construction, and if steady state pore pressure conditions exist prior to construction, then the initial excess pore pressures are zero everywhere in the foundation.

While the physical problem is three-dimensional in space and time, the spreadsheet is only two-dimensional. This limitation is overcome by using a macro to create a loop, comparable to a DO LOOP in a conventional computer program, where each execution of the loop corresponds to an increment in time (Δt). Figure 5 shows the basic spreadsheet flow diagram for the solution and may assist the reader in following the discussion. To create the loop, one range of cells in the spreadsheet is assigned the excess pore pressure values at time t, before the time step, for each of the nodal points. The cells within this range contain numerical values that correspond to the terms on the right side of Equation 10, of the form $u(x_i, y_j, t)$. Another range of cells within the same spreadsheet contains the finite difference formulas for the excess pore pressure at time $t + \Delta t$, for

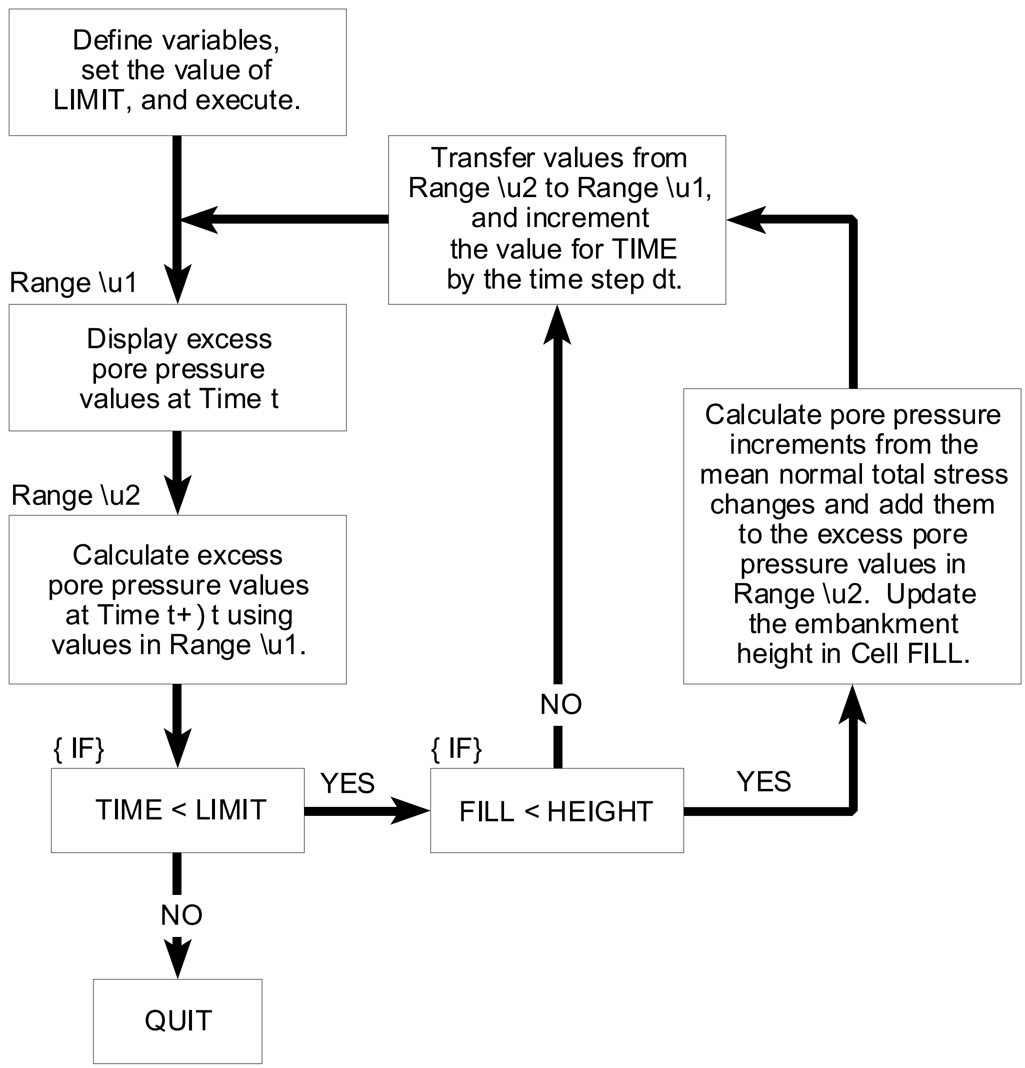

FIGURE 5: Spreadsheet Flow Diagram

each corresponding node in the first range. These formulas are developed from Equation 10, using relative cell addresses, and the cells within this range correspond to the excess pore pressure terms on the left side of Equation 10, of the form $u(x_i, y_j, t+\Delta t)$. The third range contains formulas, based on Equation 5 and the embankment geometry, that compute the pore pressure increase ($\Delta u = \Delta p$) at each nodal point arising from the application of each successive lift of material as the embankment is constructed. The use of relative and absolute cell addresses in the spreadsheet allows any given formula to be created once and duplicated as many times as necessary in the other cells of the range; however, this is a step in which significant errors can be introduced in the spreadsheet if the designer does not pay close attention to boundary conditions. In the spreadsheet for the example problem the reader should pay particular attention to the variations in the form of the finite difference expressions for those cells (nodes) that lie on no flow boundaries.

The spreadsheet sequences through the analysis as follows (the terms in brackets [ ] refer to the respective cell or range names in the spreadsheet for the example problem):

a) the lift thickness [LIFT] is computed as the product of the time increment [dt] and fill placement rate [RATE] specified by the designer;
b) the vertical stress increase [W], corresponding to the strip load (w) in Figure 2, is computed as the product of the embankment fill weight [WEIGHT] and lift thickness [LIFT];
c) the computer automatically recalculates all formulas in the spreadsheet with this new cell entry; therefore, the pore pressure increment (Δu), which corresponds to the mean normal total stress change (Δp), is calculated in the third range [\dp] for each nodal point below the current fill elevation and added to the excess pore pressure values in the second range [\u2];
d) the excess pore pressures are computed in the second range [\u2] for each nodal point, using the excess pore pressures prior to the time step from the first range [\u1] and the pore pressure increments ($\Delta u = \Delta p$) from the third range [\dp];
e) the excess pore pressures in the second range [\u2] are transferred as values into the first range [\u1];
f) the height of the fill [FILL] is incremented by the lift thickness [LIFT], and the half width (refer to Figure 3) of the next lift [B/2] is computed based on the lift thickness and side slope [SLOPE];
g) the current time [TIME] is incremented by Δt [dt], and the sequence is reinitiated until the specified time limit [LIMIT] is reached.

When execution is terminated, the excess pore water pressures displayed in the first range [\u1] are those corresponding to the current time [TIME] and embankment fill height [FILL] displayed in those respective spreadsheet cells.

In defining the size of the finite difference grid, the distances between nodal points (Δx and Δy) is quite important. Note that the values of Δx and Δy must be chosen so as to satisfy the stability and convergence criterion defined in Equation 11. The author finds it preferable to specify Δx and Δy and compute a value for the time step (Δt) that satisfies Equation 11. While Δx and Δy do not have to be equal, setting them equal simplifies Equation 10 by allowing terms to be combined as was done in the spreadsheet for the example problem. A distance of 2 to 8 feet typically provides good definition of excess pore pressures in the model; however, the example spreadsheet uses a value of $\Delta x = \Delta y = 20$ feet in order to minimize the size of the spreadsheet file while still illustrating the important points of the analysis. Thus, from Equation 11 using $c_v = 0.80$ square feet per day, the maximum time step (Δt) must not exceed 125 days for the example problem. That is a bit large for practical purposes, since the estimated time for construction is expected to be less than 60 days. By selecting a value for the time step (Δt) so as to limit the strip load (w) to a value corresponding to no more than the thickness of a construction lift, and decrementing the dimension B in Figure 2 after each time step prior to the application of the next load increment, it is possible to simulate construction of the actual embankment geometry, as illustrated in Figure 3, for any construction sequence that the designer may wish to examine. Setting $\Delta t = 1$ day corresponds nicely with the estimated fill placement rate of 1 foot per day and will more than satisfy the stability and convergence criteria. Thus, the analytical solution assumes that 12 inches of fill (a reasonable assumption for an uncompacted lift of fill material) will be placed at the end of each time step (Δt)

of 1 day, which sets the value of the vertical stress increase (w) for the uniform strip load in Figure 2 and Equation 5 at 130 psf (pounds per square foot).

As previously noted, for this example problem the increase in excess pore pressure is taken to be equal to the mean normal total stress change (Δp) at each respective nodal point, calculated using Equation 5. In computing the mean normal total stress change at all locations in the foundation, it may appear that the determination of the angle α formed between each nodal point and the base of each successive lift would be an extremely difficult task; however, closer examination of the geometry reveals it to be a simple trigonometric problem as illustrated in Figure 6. The difficulty with the angle α is that it is not part of a right triangle; however, the angle β is part of a right

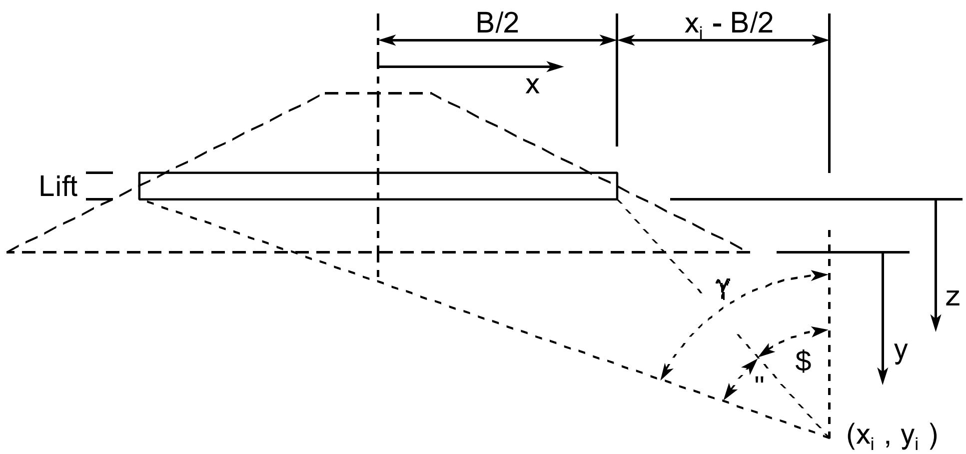

FIGURE 6: Trigonometric relations of the stress distribution model.

triangle, as is the angle γ, which is the sum of the angles α and β. Thus, the angle α can be calculated simply from the angles β and γ, which can in turn be determined from the following relationships.

$$\tan\beta = \frac{x - B/2}{z} \tag{12}$$

$$\tan\gamma = \frac{x + B/2}{z} \tag{13}$$

$$\alpha = \gamma - \beta \tag{14}$$

From Figure 6 the relations between the coordinates x, y, and z become a little more apparent. Each nodal point is fixed in space by its x and y coordinates, which remain constant during the analysis. The distance z is always measured from the bottom of the lift to the nodal point, which varies with the position of each respective lift in the embankment. For the very first lift, both z and y are the same; however, with each successive lift z is incremented by the lift thickness. It is this relationship that enables the variable z to be computed quite simply. The manner in which these variables are computed and applied to Equation 5 is illustrated in the third range of cells (\dp) in the spreadsheet for the example problem, which contains the values for the mean normal total stress change (Δp) at each nodal point for each lift applied to the embankment. The reader should pay particular attention to the manner in which the variables B/2 and LIFT are computed in the spreadsheet. The cell containing the variable B/2 contains a simple algorithm that computes the parameter based on the geometry of the embankment and the position of each respective lift, which is tracked by the variable FILL in the example spreadsheet. The value of the variable LIFT is the product of the fill placement rate [RATE] (which was determined to be approximately 1 foot per day for the example problem) and the time step [dt] (which was chosen to be 1 day for this problem). A conditional statement has been placed in the cell containing the variable LIFT that compares the value of the variable FILL to the completed embankment height [HEIGHT] and sets the value of LIFT to zero once the embankment height has been reached in the model.

The actual macro commands that execute the analysis are shown in red to the right of the variables at the top of Sheet 3 in the example spreadsheet; however, the actual macro resides in a Visual Basic Module. Pressing the keys [Alt] and [F8] will open the dialogue box for the module. The macro in the example spreadsheet was given the name "Execute," and clicking on the EDIT button opens the macro. The reader may note that the use of range names makes the logic used in the macro commands much easier to understand. In addition, Excel allows the reader to insert comment statements (which appear in green) anywhere within the macro itself. The second command in the macro, "Range("\u1").Value = Range("\u2").Value," copies the results of the equations in Range \u2 as values into Range \u1, which lies at the heart of the spreadsheet model.

The use of a conditional statement in the cell containing the value of the variable LIFT also allows the designer to model stage construction if the stability analyses indicate the need to allow for some dissipation of excess pore pressures before the embankment can be built to its complete height. The following illustrates this approach for the example problem. Let us assume that the embankment will first be constructed to a height of 30 feet (which would require 30 days in this model). Construction will then be halted for 180 days, or approximately 6 months, after which fill will be placed until the embankment is completed (which will require 30 more days). Placing a conditional statement of the following form in the cell for the variable LIFT will simulate this sequence of events.

```
=IF(FILL<HEIGHT, IF(TIME<=30,RATE*dt, IF(TIME>210, RATE*dt, 0)),0)
```

Observe the values in Range \u1, LIFT, and FILL as the spreadsheet executes this analysis. FILL varies from 0 to 30 feet in the first 30 days, remains constant at 30 feet from 30 to 210 days, increases to 60 feet from 210 to 240 days, and remains constant for any time greater than 240 days.

Note that the solution employs a forward marching technique, which is to say that the analysis can be stopped at any point by simply specifying the value of LIMIT; however, the value must always be increasing in time. For example, in the case of the stage construction previously described the designer can set the value of LIMIT at 30 days and run the analysis. The value can be reset to 210 days, the macro rerun, and the spreadsheet will continue the analysis up to the specified value of LIMIT. Similarly the value of LIMIT can be reset to 240 days, 365 days, etc.; however, the LIMIT cannot be set to one value (say 365 days), the macro executed, and the LIMIT reset to a lesser value (such as 210 days). Just as in the real world, time only moves in the forward direction. However, resetting the spreadsheet for the example problem is a very simple process. Simply delete all the values in Range \u1 (which has the same effect as setting them to zero), reset the values in cells FILL and TIME to zero, and specify the desired value of LIMIT.

LIMITATIONS

While the author believes that the approach presented in this paper is a potentially useful method for analyzing the problem of construction generated pore pressures, it is important for the reader to be aware of the limitations inherent in this approach. It is hoped that this will not only assist the designer in correctly applying the method, but also in interpreting any field data collected during project construction.

As previously noted, Equation 7 oversimplifies the relationship between the change in pore pressure and the change in mean normal total stress by ignoring the actual stress-strain properties of the consolidating soil, which the ‘a’ parameter attempts to address. However, using Equation 4, which incorporates the ‘a’ parameter and stress invariant q, is by no means as simple as it may appear. A major reason for this is the matter of selecting an appropriate value for the ‘a’ parameter, which is not a constant, but rather changes during loading and the consolidation process. Furthermore, the error in the result is compounded when both the p and q terms are used due to the inherent errors in the estimated values of σ_x, σ_z, and τ_{xz} needed to compute the stress invariants p and q.

The boundary conditions associated with Equations 1 through 3 are violated for all lifts except the very first one, because the equations assume that the strip load is placed at the surface of a homogeneous, isotropic, linear elastic, half space. The error, however, is likely to be relatively small for the case discussed in this paper. As previously noted, the model was originally developed by the author to analyze the problem of excess pore pressure development within the core zone of an earth and rockfill dam during construction, and the results obtained from the model agreed reasonably well with pore pressures actually measured in the field. Nevertheless, a comparison of the stresses computed using the simple elastic model against those obtained from a finite element analysis (which is also only a numerical model) found very good agreement for vertical stresses, while the horizontal stresses computed using the simple model were about twice as large as those obtained from the finite element analysis. The relative difference in the horizontal stresses seems appropriate, since the soil is free to strain in the horizontal direction within an embankment, while not within the half-space on which the equations are predicated. Consequently, one would expect the difference between the actual and predicted horizontal stresses to be even less for the case of a foundation, which more closely approximates a half-space, despite the fact that the strip loads are not placed at the foundation surface.

The issue of estimating the stress change in soil that will occur in response to an applied load is perhaps the greatest limitation of all solutions to problems in consolidation and settlement. In the real world it is rare to find a geologic deposit that is even relatively homogeneous, let alone isotropic. It has long been recognized that soils are not totally elastic, though their behavior may be relatively elastic within certain stress ranges. Consequently, we must accept that our methods can only be considered approximate when applied to stratified deposits of soils with differing stress-strain properties. The problem of stratified soil deposits has been addressed by Burmister, Westergaard, and others; however, even these methods assume that the individual layers behave elastically. Even a finite element solution should be considered approximate, since the values for the Elastic Modulus and Poisson’s Ratio are not only difficult to determine, but actually change during the consolidation process. Therefore, in the author’s opinion, our ability (or inability) to predict the stress changes that will occur in the ground in response to a surface load is perhaps the greatest limitation in our current solutions to the problems of consolidation and settlement.

It is relevant to note here that the issue of the soil's actual stress-strain properties and their effect on the excess pore pressures that develop in response to load, as well as the effect of the soil's initial degree of saturation are limitations that are not as difficult to overcome. The author's second paper on the subject of end of construction pore pressures within the embankment itself describes a method for selecting values of the coefficient 'b' in Equation 7 that endeavors to account for the soil's stress-strain properties and initial degree of saturation based on the work of Hilf [3], which uses the results of the standard one-dimensional consolidation test.

Finally, it is very important to recognize the effect on the results that the consolidation coefficient (c_v) has, which is also changing throughout the foundation during the consolidation process. The work of Lo, et al. [8] revealed that consolidation tests on 2 inch diameter samples underestimated the coefficient of consolidation by one to two orders of magnitude compared to tests performed on 6 inch diameter samples. The work of Leonards and Girault [7], using samples approximately 4.4 inches in diameter, revealed that both the load increment ratio and side friction between the sample and consolidometer ring could have a very significant effect on the rate of excess pore pressure dissipation during the test. As is the case with any type of analysis, the results are only as good as the data. Fortunately computer models lend themselves well to sensitivity analyses and the spreadsheet is no exception. Once the spreadsheet model has been built it is a simple matter to rerun the analysis for different values of the coefficient of consolidation to establish potential maximum and minimum values for the excess pore pressures.

CONCLUDING REMARKS

A simple model has been proposed for estimating excess pore pressures generated in a fully saturated soft soil foundation in response to an applied surface load. The model provides a method for considering the stress distribution within the foundation, along with a two-dimensional finite difference algorithm that addresses the simultaneous generation and dissipation of excess pore pressures that occur during construction. The primary advantage of the model is that it involves relatively simple algorithms that lend themselves well to solution using commercially available spreadsheet software run on a personal computer. Despite its limitations, pore pressures estimated using this simple model have been found to agree reasonably well with actual values observed during the construction of a well instrumented large zoned earth and rockfill dam, with the error being on the conservative side: that is, the actual observed values were less than the predicted values.

It is suggested that the method proposed herein be applied with care and good judgment to the design of projects that are sufficiently instrumented to ensure that excess pore pressures generated during construction do not exceed the values predicted by the model. While the method shows promise as a design tool, additional comparisons should naturally be made between the proposed model and actual field data collected from other projects. A larger body of comprehensive field data, consisting of total stress, pore pressure, and settlement measurements, will be required to develop a better understanding of actual soil behavior, which will ultimately lead to safer, more cost effective designs. Though the primary purpose of geotechnical instrumentation should be to verify the design assumptions and identify potential problems before the structure experiences significant distress, the data collected to meet these objectives also provides the best means of developing a better understanding of the actual behavior of soil foundations and earth structures. Since clients will naturally be reluctant to finance what they may perceive as research, the designer must communicate the importance of instrumentation to the client by pointing out that a more cost effective design can be achieved if the need for overly conservative assumptions can be eliminated; however, this requires verification of actual performance in the field. To maximize the effectiveness of the instruments, the project should also call for index property tests of the material in the immediate vicinity of the instruments, as well as shear strength and consolidation tests of the respective materials in the foundation and/or embankment, which will provide the information necessary for application to analytical models.

The author would sincerely appreciate any suggestions or comments on this paper and/or the spreadsheet for the example problem from those engineers who attempt to use this model. The author would also be particularly interested in the details of any projects where instrumentation data collected in the field has been or is being compared to values predicted using the model. As a final note, the author strongly urges individuals not to attempt to modify the example spreadsheet to fit any problems on which they are working, but rather to develop their own spreadsheet solution. Creating your own spreadsheet not only helps you to better understand the underlying technical principles, but also minimizes the potential for errors that frequently arise when attempting to use someone else's spreadsheet.

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to Ms. Marie J. Starich. P.E., of Solum Engineering, Inc., Houston, Texas; Mr. Jeffrey B. Fassett, P.E., of Golder Associates Inc., Houston, Texas; and Dr. Dobroslav Znidarcic of the University of Colorado at Boulder, for their time in reviewing this paper and associated spreadsheet, as well as for their valuable comments and suggestions.

DISCLAIMER

The spreadsheet solution to the example problem is made available over the Internet as a companion document to the paper titled "A Simple Model For Estimating End Of Construction Pore Pressures: Part 1 - Foundation Pore Pressures," by David J. Kerkes, © 2001. The spreadsheet provides a solution to the example problem discussed in the paper and is intended for that purpose and no other. This spreadsheet is not intended to serve as a template for use by individuals in the development of solutions to their own specific problems, but rather as an example to assist individuals in developing their own models. While considerable effort has been spent to ensure that the spreadsheet is free of errors and computer viruses, the author does not warrant that the spreadsheet is error or virus free. Individuals should be particularly cautious when using copies of the spreadsheet not obtained directly from the author's web site. The decision to place any reliance on the method illustrated in this spreadsheet, as well as any conclusions drawn from the results, are ultimately the responsibility of the designer.

The date is provided on Sheets 1 through 4 of the spreadsheet and individuals may wish to periodically visit the author's web site for updated versions of the paper and/or spreadsheet.

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Appendix A

The following provides the intermediate steps in the development of the expressions for the stress invariants p and q under plane strain conditions, given by Equations 5 and 6 respectively, in the section titled **PORE PRESSURE RESPONSE IN SOIL**.

We begin with the definition of the total stress invariants p and q [1]:

$$p = 1/3\ [\sigma_z + \sigma_x + \sigma_y] \tag{A1}$$

which is simply the mean or average normal total stress, and

$$q = (1/\sqrt{2})[(\sigma_z - \sigma_x)^2 + (\sigma_z - \sigma_y)^2 + (\sigma_y - \sigma_x)^2 + 6(\tau_{xz}^2 + \tau_{yx}^2 + \tau_{yz}^2)]^{½} \tag{A2}$$

Noting that under plane strain conditions $\varepsilon_y = 0$, and applying the equation of elasticity for ε_y yields the following:

$$\varepsilon_y = 0 = 1/E(\sigma_y - \mu\sigma_z - \mu\sigma_x)$$

solving for σ_y yields:

$$\sigma_y = \mu(\sigma_z + \sigma_x) \tag{A3}$$

Recall Equations 1, 2, and 3 for σ_z, σ_x, and τ_{xz} respectively (as previously defined in the **STRESS DISTRIBUTION MODEL** section):

$$\sigma_z = (w/\pi)[\alpha + \sin\alpha\cos(\alpha + 2\beta)] \tag{1}$$
$$\sigma_x = (w/\pi)[\alpha - \sin\alpha\cos(\alpha + 2\beta)] \tag{2}$$
$$\tau_{xz} = (w/\pi)[\sin\alpha\sin(\alpha + 2\beta)] \tag{3}$$

Now substituting Equation A3 into Equation A1 with Poisson's ratio (μ) equal to 0.5 for undrained loading of a saturated soil, and using Equations 1 and 2 for σ_z, σ_x, yields the following expression for the total stress invariant p (mean normal total stress).

$$p = w\alpha/\pi \tag{5}$$

Similarly noting that under plane strain conditions $\gamma_{yz} = \gamma_{yx} = 0$, and applying the equations of elasticity for τ_{yz} and τ_{yx}, yields the following:

$$\tau_{yz} = G\,\gamma_{yz} \quad \text{and} \quad \tau_{yx} = G\,\gamma_{yx} \quad \text{therefore,} \quad \tau_{yz} = \gamma_{yx} = 0 \tag{A4}$$

In like manner by substituting into Equation A2 the respective expressions for the normal and shear stresses from Equations 1, 2, 3, A3, and A4, with Poisson's ratio again equal to 0.5 for undrained loading of a saturated soil, the following expression for the total stress invariant q is obtained.

$$q = (w/\pi)(\sin\alpha)(\text{o}3) \tag{6}$$